On Some Geometrical Aspects Of Land Surveying In The Middle Ages

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INTRODUCTION

As to the fact that quite a number of publications deal with the innovations in geometry and mathematics on the verge of early modern times,[1] but only few discuss land surveying during the Middle Ages, this paper is dedicated to the explanation of some of the main sources of errors made in land survey during this epoch, as witnessed by contemporary authors, and to show how farfetched the knowledge on some aspects of land surveying was.

Literature on the subject is scattered and quite a few studies are found in publications out of reach of the historian. Klaus Grewe has recently compiled a bibliography on the subject.[2] In general, two fields have drawn the attention of research: First, the historical point of view, coping with the determination of the actual size of old measures in certain regions,[3] or with metrological problems mainly.[4] Secondly, geodetic surveyors dealing with historical subjects concentrate highly on mere technical details of certain instruments. Besides that, if they deal with theoretical aspects in the historians' field, the outcoming studies are in some cases not meeting the historians' quality standards.[5]

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[1] See, as an example, the studies on Regiomontanus' life and work, recently, e. g., Günther *Hamann* (ed.), Regiomontanus-Studien (= Veröffentlichungen der Kommission für Geschichte der Mathematik, Naturwissenschaften und Medizin 28,29,30) Vienna, 1980.

[2] Klaus *Grewe*, Bibliographie zur Geschichte des Vermessungswesens (= Schriftenreihe des Förderkreises des Vermessungstechnischen Museums) Cologne, 1984.

[3] Walter *Kuhn*, Vergleichende Untersuchungen zur mittelalterlichen Ostsiedlung (= Ostmitteleuropa in Vergangenheit und Gegenwart) Cologne, Vienna, 1973, p. 150 f.

[4] Fritz *Bönisch*, Zur Auflösung von Vermessungsangaben in geschichtlichen Quellen. In: Heimatkunde und Landesgeschichte, ed. Friedrich *Beck* (= Veröffentlichungen des Brandenburgischen Landeshauptarchives 2) Weimar, 1958, p. 135-150.

[5] An example for this is Paul *Stichling*, Die Pflege der Geschichte des Vermessungswesens I. In: Zeitschrift für Vermessungswesen 74 (1949) 91. More recently: Volker *Bialas*, Die Geodäsie und ihre Geschichte - wissenschaftstheoretische Aspekte (= Geschichte und Entwicklung der Geodäsie, E 22) Munich, 1984, p. 23, ann. 1.

Astronomy, although using the same instruments,[6] deals with different problems, as does navigation.[7]

SOURCES

For this study two sources have been selected: First, the "Geometria Culmensis",[8] written between 1393 and 1407 by order of Conrad von Jungingen, being Grand Master of the Teutonic Order at that time. It was chosen due to the fact that it is one of the very few sources appearing not only in Latin, but as well in Middle High German, whereas, e. g., the geometrical tractate of Robertus Anglicus, written in 1271, was translated only in 1477.[9] A treatise written in the vernacular may lead to the assumption that it was dedicated to practical use. The second source discussed in this study is the "Practica Geometriae", dating from the end of the 12th century.[10]

A study of land surveying in the Middle Ages cannot leave out Gerbert's works on the subject. Werner Bergmann[11] points out in his study that Gerbert was using land surveying as a perfect example for both geometry and astronomy, not dealing in particular with the problems arising during practical work. It seems important though, that Gerbert was obviously using the "Corpus Agrimensorum Romanorum" as source for his own studies.[12]

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6 See, e. g., Fritz *Schmidt*, Geschichte der geodätischen Instrumente und Verfahren im Altertum und Mittelalter (= Veröffentlichungen der Pfälzischen Gesellschaft zur Förderung der Wissenschaften 24) Neustadt/Haardt, 1935, p. 22.

7 The main problem of navigation is the localisation of a moving point on a map grid. For this, exact measurement of time is absolutely necessary, as the degree of longitude cannot be measured otherwise.

8 Edited in: Urkunden und erzählende Quellen zur deutschen Ostsiedlung im Mittelalter (= Ausgewählte Quellen zur deutschen Geschichte des Mittelalters I, Fr. v. Stein Gedächtnisausgabe XXVI a), ed. Herbert *Helbig*, Lorenz *Weinrich* (Darmstadt, 1968) p. 524–531, n. 143.

9 *Schmidt*, Geschichte der geodätischen Instrumente, p. 24, ann. 69.

10 Maximilian *Curtze*, Practica Geometriae. Ein anonymer Traktat aus dem Ende des zwölften Jahrhunderts. In: Monatshefte für Mathematik und Physik VIII (1897) p. 193–224.

11 Werner *Bergmann*, Gerbert von Aurillac und die Landvermessung. In: Ingenieurvermessung von der Antike bis zur Neuzeit 3, Symposium zur Vermessungsgeschichte in Dortmund 16. 2. 1987, ed. Hartwig *Junius* (Dortmund, 1987).

12 *Bergmann*, Gerbert, p. 109 and 138, ann. 12.

For the various other sources on land surveying the reader is asked to consult Fritz Schmidt.[13] Generally, sources tend to deal more with the theoretical aspects of the problem, as, e. g., determination of the earth's circumference, than with the practical problems. This is also shown by Volker Bialas,[14] who, although dealing with early modern times mainly, has been pointing out that during the Middle Ages theoretical problems absorbed the scientists' thought. Even after the beginning of the seafaring age, when the interest in problems of navigation and cartography increased, sources are dealing with purely geometrical problems more than with problems of what we would nowadays call applied science. As one of the very few examples for the opposite, the "Tractatus Geometriae Practicae" by Martinus Polonus[15] should be mentioned here.

Solutions of Survey Problems in the Middle Ages

Theory

Distance measurement is in fact a comparison of two distances: The one you know (your measure) and the one you want to know. This is also true for area measurement, where you compare with an area unit which gives the area of a plane piece of land by multiplication.

Although this sounds easy, and is in fact theoretically easy, various sources of errors can be found. In the Geometria Culmensis we find on the subject: *Nec oportet ex hoc superficiem mensurandam quadratam esse, quod mensuratur per superficies quadratas, ymmo triangulus et poligonia et superficies irregulares et circulus mensurantur superficie quadrata. Ideo dixi, quod oportet, superficiem famosam et notam esse quadratam, que debet esse mensura aliarum superficierum, cuiusque dispositionis existant.*[16] It had to be explained that the square could be used as a standard for area determination of circular or else shaped pieces.

Determination of heights and depths not accessible to direct measurement present one of the problems, sources frequently deal with. The length of a distance not completely in sight of the surveyor (e. g. on a hill) gives the second one. I will only deal with planimetric problems here.

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[13] *Schmidt*, Geschichte der geodätischen Instrumente, passim.

[14] Volker *Bialas*, Praxis Geometrica. Zur Geschichte der Geodäsie am Beginn der Neuzeit (= Geschichte und Entwicklung der Geodäsie E 11) Munich, 1970.

[15] Mag. Martini de Zorawica alias "Martinus Rex de Premislia" vocitati Geometriae practica seu artis mensurationem tractatus, ed. L. *Birkenmajer* (Warsaw, 1895).

[16] Geometria Culmensis, p. 528.

It has to be stressed that the shape of the piece of land to be surveyed is most important. In reality, we will come upon irregular forms a lot more often than upon perfect squares and rectangles. For irregular polygons determination of angles is necessary for area measurement. All these problems are independent of the actual size of the measure used.

Practical Aspects

For practical survey the unit of measure is of central importance. Standardization of measures for length and area is a difficult task, as can be judged from the 19th century work on the "meter-standard" which was prepared with the utmost accuracy possible, but had no theoretical foundation. Only nowadays a theoretical basis which is not subject to any possible change has been created for the meter by using the wavelength radiated by a defined atomic process. No wonder that the medieval society had its problems with standardization as well. Units of length were primarily taken from the only standard accessible to everyone everywhere: the human body. The Geometria Culmensis is a vivid example for standardization on human measures, saying ... *in der Handbreit steckt viermal der Finger, viermal im Fuß die Hand ...*[17] Medieval sources show that people were aware of the bias due to differences of this measure,[18] but accepted the bias, as the availability of the measure was the greater opportunity compared to its stability.

Human measures, apart from their variation present another problem to the surveyor: Their size is very small compared to the sizes of the pieces of land to be measured. Direct comparison would have been extremely tiresome and erratic. Strings with knots in regular intervals could be used for the measurement of larger distances. As they were made out of natural fibres, large differences in their actual measure occurred depending on the relative air humidity or on the wetness of the ground they were laid upon. Metal chains, which are described in the Geometria Culmensis as well as the strings, were not susceptible to this bias, but were heavy and thus unpracticable for transport over large distances, additionally representing quite some value at that time.[19]

All these instruments, as well as the pole, which was quite frequently used even later, as the experimental determination of its length by averaging the foot measure of 16 chosen people shows,[20] have one major

[17] Geometria Culmensis, p. 531.

[18] Urkunden und erzählende Quellen, p. 524, n. 142.

[19] *Bialas*, Praxis Geometrica, p. 4.

[20] *Bialas*, Praxis Geometrica, p. 2, reporting a procedure described by Köbel in 1522.

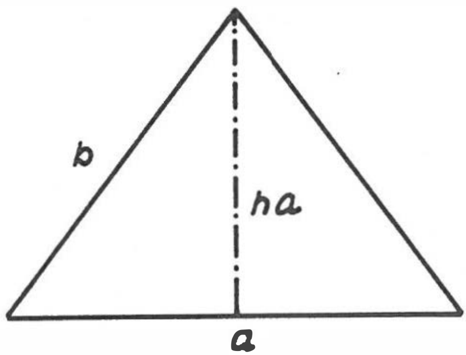
disadvantage: They imply direct access to the distance to be measured. This system has its limitations in the measurement of large extensions, as were usual in land survey. Estimation of usual sizes is possible from the definition of the "Joch", which was the area which could be ploughed with a yoke of oxen in one day. For a study on the development of this measure, see the publication of Egghart.[21]

In the following, some examples for the solution of geometrical problems in the sources are given to illustrate the medieval standard of knowledge.

The Geometria Culmensis gives the right formula for the area of a triangle, mentioning that taking the length of a side of a triangle instead of the height (as shown in Fig. 1) causes errors in measurement.

Fig. 1: Area of a triangle

Formula: $A = a * h(a)/2$
Erratic medieval version: $A^* = a * b/2$
In this case: $a = 60, b = 50, h(a) = 40$
$A = 60 * 40/2 = 1200mm^2$
$A^* = 60 * 50/2 = 1500mm^2$
Error: A^* amounts to 125% compared to A



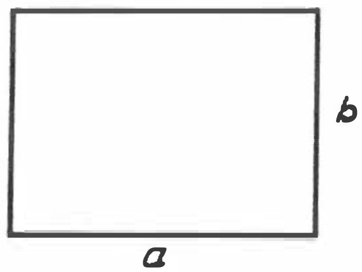
[21] Kurt *Egghart*, Das österreichische Joch und seine keltisch-gallische Wurzel. In: Österreichische Zeitschrift für Vermessungswesen 53 (1965) p. 153–162.

Errors due to implementation of (b) – one side of a triangle – instead of h(a) – height of triangle over the baseline – can amount to several percent of the area to be measured, depending on the shape of the triangle.

One might wonder why triangles are of such importance to the unknown author of the Geometria Culmensis, as we usually find rectangular pieces of land. Most pieces are not rectangular in fact, and those irregular polygons can only be measured by dividing them into triangles. It was quite usual, even in the 16^{th} century, as discussed by Bõnisch,[22] to calculate the area of irregular pieces with four sides by averaging two and two of them and treating those averages with the formula applicable to rectangles (see Fig. 2).

Fig. 2: Polynom area determination

Rectangle: $A = a * b$

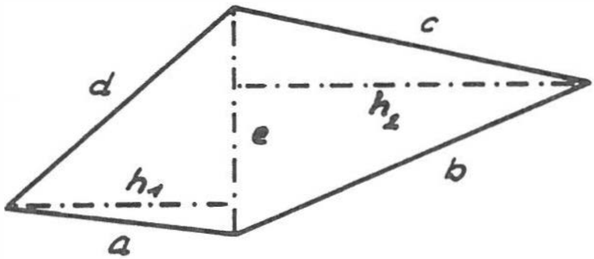


Polynoms were treated with the same formula using the arithmetic means of two sides:
$A^* = (a + c)/2 * (b + d)/2$

Correct results are obtained by splitting the polynom into triangles using the area determination according to Fig. 1.

In this case: $a = 30, b = 40, c = 48, d = 50$
$A^* = ((30 + 48)/2) * ((40 + 50)/2) = 1755mm^2.$
Correct calculation yields $(e = 30, h_1 \approx 30, h_2 = 47)$*:*
$A = 30 * 30/2 + 30 * 47/2 = 1155mm^2$
Error: A^* *amounts to 152% compared to A.*

[22] *Bõnisch*, Zur Auflösung, p. 141 f.



For the 14th century Kuhn[23] notes that the surveyors tried to mark out pieces of land almost rectangular and regularly shaped, which might have to do with the easiness of surveying in such cases.

The measuring instruments were known to Gerbert and are described in detail by Schmidt.[24] The commonly known instruments were the gnomon and a cross-like shaped piece called *crucze* in the German version of the Geometria Culmensis. For both instruments two ways of application were possible: they could be put on the earth's surface and the distances to be measured would then be measured with strings spanned in the directions indicated by the cross, or they were used for aiming.

Aiming is the most efficient means of minimizing the expenses of surveying procedures. Two mathematical laws are implied in this procedure: First the similar rectilinear figures theorem, describing such figures as having their several angles equal each to each, and the sides about the equal angles proportional. This not only provides a tool for the measurement of distances out of reach (as, e. g., the width of a river[25]), but presents a possibility of surveying a very large area by building a figure with the same proportions out of twigs or similar material, which then could be measured much easier.[26] Fritz Schmidt explains the use of an instrument for this purpose, made of three sticks with regular marks of which two were connected on one end, opening up a variable angle. The third stick was used to adjust the shape of the construction according to the proportions of the piece of land to be measured. The advantage then was not only the lessened practical expense, but also the

23 *Kuhn*, Untersuchungen, p. 64 and 150.

24 *Schmidt*, Geschichte der geodätischen Instrumente, p. 115.

25 *Bergmann*, Gerbert, p. 123.

26 *Kuhn*, Untersuchungen, p. 150.

avoidance of any calculation,[27] as only the unit of measurement had to be changed and the marks could easily be counted.

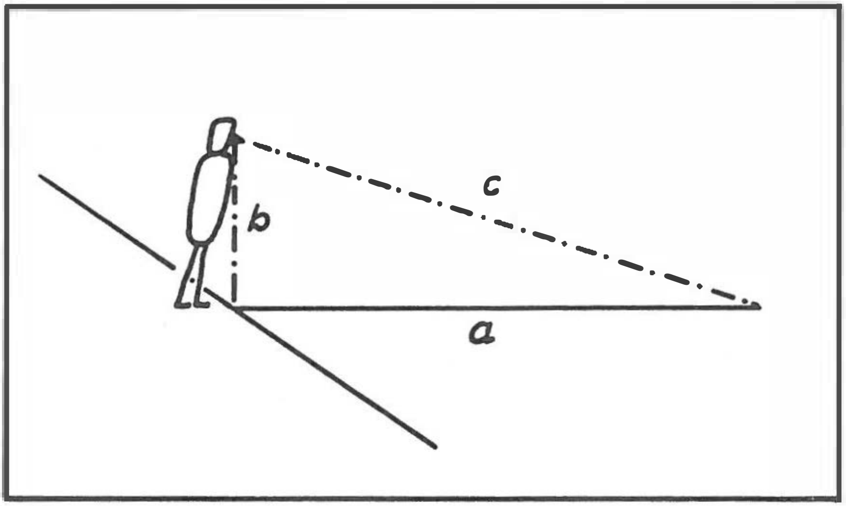
With the second example it can be shown that, in spite of the description in Gerberts oeuvre written almost 400 years ahead of the Geometria Culmensis and the detailed remarks on the subject in the Practica Geometriae two centuries earlier, the author of the Geometria Culmensis still had to complain about the errors made in the calculations implied in aiming. Fig. 3 shows the principal problem.

Fig. 3: Aiming

The height of the surveyor has to be taken into calculation, following:
$a^2 + b^2 = c^2$, *thus* $a = \sqrt{c^2 - b^2}$

In this case: $a = 70$, $b = 24$, $c = 74$
$a = \sqrt{74^2 - 24^2} = 70$

The error of approx. 5% in length is multiplicated during area calculation, leading to much greater errors.



[27] *Schmidt*, Geschichte der geodätischen Instrumente, p. 372.

In aiming you start making errors in the moment you stand up, instead of working as near to the surface as possible. As in the river width example the height of the surveyor has to be included in the calculation, this time by application of the Pythagorean theorem, which was known to the authors.

The Geometria Culmensis gives a good approximation for the square root calculation, which was needed in the Pythagorean theorem by splitting the number into two parts, following:

$$\sqrt{x} = \sqrt{a^2 + b} \approx a + b/2a$$

where a is the nearest square number. For x = 250, 225 fulfills this prerequisite, being the square of 15, thus $\sqrt{250} = 15 + 25/2 * 15$ resulting in $\sqrt{250} = 15.833$. A comparison to the calculated number $\sqrt{250} = 15.811$ shows that the deviation is only 0.14 %, a negligible amount.

Even better agreement with calculated values is achieved in the approximation given for the number π, in this case cited from the Practica Geometriae, as it follows the older sources (e. g. Gerbert) as well as the Geometria Culmensis:[28] π is approximated by 22/7, which gives a value of 3.1428, compared with the real value of 3.1416 making up a difference of 0.04 %. This deviation is of no interest in practical measures, even if values as big as the earth's circumference are to be calculated with this approximation.

It was mentioned above that the surveyor could adjust the shape of the pieces to be surveyed according to his possibilities if new land was to be cultivated. A slightly different sort of problem was given in cases, where existing areas were to be sized. One of the questions soluble only by application of the Pythagorean theorem is the question, if a piece of land was rectangular or not. This was of particular interest not for cultivation lands, but for places where buildings were to be erected. As the matter is discussed in Fritz Schmidt's work,[29] I will only briefly comment on it:

Rectangular triangles show certain proportions in the length of their sides, as described by Pythagoras. If one takes a piece of string with marks and constructs a triangle with such proportions one then has constructed a 90° angle as well. This is useful for determining rectangularity of existing structures as well as for constructing them.

[28] Practica Geometriae, p. 216.

[29] *Schmidt*, Geschichte der geodätischen Instrumente, p. 98–101.

Remarks on the importance of land survey in the medieval society

Surveying is unlike other scientific developments arising from practical needs of the people. Fritz Schmidt puts forward the building of houses and the transport of water through tubes as the origin of surveying.[30] Although this is true, the real need for developing land survey methods definitely has been the determination of measures of cultivated lands, as distance measurement itself does not imply area calculation which has been the driving element of theoretical development. No wonder that in Egypt earliest sources have been found, as the yearly floods of the Nile made survey absolutely necessary.

Ancient Egypt fulfilled the second requirement for the development of land surveying much better than the Middle Ages did: A certain degree of organisation is necessary, as you need to have registration of measures and education of personnel. The 'Vogt' of the county of Oberschwaben, e. g., left the charter, which he had given concerning the width of a street, in the nearby Cistercian abbey of Salem, as he had no measure registration of his own. This may serve as an example for the low degree of administrative development at that time.[31] Routine measurements could not be performed under such circumstances.

The Geometria Culmensis shows which kind of social background enhances land survey studies: The Teutonic Order, having a high degree of inner organisation, cultivated large areas for the first time, thus the surveyors could determine the shape according to their needs. The need to survey large areas encouraged the development of indirect measurement methods as described above. Walter Kuhn[32] gives details on the organisation of the Teutonic Order, but his statement that the three-field-system of land cultivation enhanced surveying lacks sound explanation. He has the equally far-fetched argumentation that a correlation between the increased corn production and the land surveying methods exists. In opposition to his findings it has to be stated that the actual measure of a field was of less importance to the possessor than its fertility. Thus measures were relatively unimportant, if the difference in fertility was high.

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[30] *Schmidt*, Geschichte der geodätischen Instrumente, p. 24, ann. 69.

[31] Heinz *Thomas*, Deutsche Geschichte des Spätmittelalters 1250–1500 (Stuttgart, Berlin, Cologne, Mainz, 1984), p. 44.

[32] Walter *Kuhn*, Neue Beiträge zur Schlesischen Siedlungsgeschichte (= Quellen und Darstellungen zur Schlesischen Geschichte 32) Sigmaringen, 1984.

Nevertheless, the re-development of land survey was enhanced by the colonisation of large areas as well as the increasing administrative level and furthermore by the beginning of seafaring and the reappearance of antique sources, many of them coming through the arabic scientists' work.

Acknowledgments:

The idea to this study, as well as my first German version of it, originate from a course in medieval history held by Herwig Weigl of the "Institut für Österreichische Geschichtsforschung", Vienna. His guidance is gratefully acknowledged. I also owe a lot to Christina Rosenberg, who has been giving me many helpful hints concerning English translations of scientific papers.

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Vorwort

Im letzten, Dänemark gewidmeten Heft hat *Medium Aevum Quotidianum* begonnen, sich mit den in einzelnen Ländern durchgeführten Forschungen zu Alltag und materieller Kultur des Mittelalters auseinanderzusetzen. Diese begonnene Reihe wird, wie angekündigt, fortgesetzt: die Vorbereitungen für ein Heft, dessen Gegenstand die Situation in Finnland sein wird, sind soweit gediehen, daß im Herbst mit dem Erscheinen zu rechnen ist. Gleiches gilt für ein Heft, welches die diesbezügliche Lage in Ungarn zum Inhalt haben wird. Kontakte mit schwedischen Kollegen zur Realisierung eines ähnlichen Vorhabens wurden aufgenommen.

Der zweite Schwerpunkt der Veröffentlichungstätigkeit wird auf der Fortführung von Auswahlbibliographien zu Teilbereichen der Geschichte von Alltag und materieller Kultur des Mittelalters liegen. In den nächsten Heften werden wir eine auf Ungarn bezogene Bibliographie vorlegen können, sowie eine solche, deren Schwerpunkt auf Südosteuropa liegt. Ferner wird die bereits angekündigte Auswahlbibliographie zu "Kleidung und Mode in Mittelalter und früher Neuzeit" zur Publikation kommen. Schließlich ist für 1990 mit dem Erscheinen einer Auswahlbibliographie zum Thema "Migration in Mittelalter und früher Neuzeit" zu rechnen sowie einer kommentierten Auswahlbibliographie zum mittelalterlichen Schiffswesen.

Das vorliegende Heft soll einerseits vor allem der Information unserer Mitglieder dienen. Einige aktuelle Ankündigungen sowie die aktualisierte Mitgliederliste unserer Gesellschaft sind in diesem Sinne zu verstehen. Andererseits widmet sich der veröffentlichte Beitrag zur Landvermessung im Mittelalter einem Themenbereich, der unseres Erachtens nach von großer Wichtigkeit für unser Forschungsfeld ist, jedoch in der wissenschaftlichen Aufarbeitung einigermaßen vernachlässigt wird. Der Autorin sei für die zur Verfügung Stellung der Arbeit gedankt.

Die Vorbereitungen für die von *Medium Aevum Quotidianum* in Zusammenarbeit mit dem *Institut für mittelalterliche Realienkunde Österreichs* der Österreichischen Akademie der Wissenschaften für 1990 in Krems geplanten internationalen Veranstaltungen sind weit gediehen. Vom 9. bis zum 12. Oktober 1990 wird der Kongreß "Alltag und Kommunikation im Mittelalter" stattfinden. Ein Teilbereich dieses Themenkreises wird ausgekoppelt und im Rahmen eines Round-Table-Gespräches am Montag, dem 8. Oktober 1990, zur Sprache kommen. Es handelt sich dabei um die Problematik von "Alltag und Wallfahrt im Mittelalter". Die Liste der jeweils vorgesehenen Referenten bzw. Teilnehmer sowie die zu behandelnden Themen stehen zum Großteil fest. Ein konkreteres, vorläufiges Programm werden wir im nächsten Heft unseres Newsletters mitteilen können. Die endgültigen Programme werden Ende des Jahres zum Versand kommen.

Die Vorbereitungen zur Publikation der Ergebnisse der Veranstaltungen des Jahres 1988 schreiten ebenfalls voran. Der Band zum Round-Table-Gespräch "Religiöse Stiftung und materielle Kultur im Mittelalter" wird in Kürze in Druck gehen und noch 1989 erscheinen. Der Band des Kongresses "Mensch und Objekt im Mittelalter. Alltag – Leben – Kultur" wird in der zweiten Jahreshälfte in Druck gehen können und 1990 vorliegen. Beide Bände werden als *Veröffentlichungen des Instituts für mittelalterliche Realienkunde Österreichs* 12 bzw. 13 im Rahmen der *Sitzungsberichte* der philosophisch-historischen Klasse der Österreichischen Akademie der Wissenschaften zur Publikation gelangen.

Das *Institut für mittelalterliche Realienkunde Österreichs* feierte vor einigen Wochen die ersten zwanzig Jahre seines Bestehens. Aus diesem Anlaß fand nicht nur ein kleiner Festakt statt, sondern es wird darüber hinaus auch eine Bibliographie aller Publikationen zur Veröffentlichung gelangen, die seit dem Bestehen aus und im Rahmen der Institutsarbeit entstanden. Ergänzt wird diese Bibliographie durch den Abdruck des Festvortrages von Wolfgang Brückner, Würzburg, zum Thema "Der Blumenstrauß als Realie. Gebrauchs- und Bedeutungswandel eines Kunstprodukts im christlichen Kult seit dem Mittelalter".

Eine besonders enge Zusammenarbeit entwickelt sich augenblicklich unter der Ägide von *Medium Aevum Quotidianum* zwischen ungarischen und österreichischen Forschern. Daraus ergibt sich nicht nur die Realisierung der oben angeführten Publikationsvorhaben, sondern eine weit darüber hinaus gehende Kooperation. Diese beinhaltet regelmäßige Treffen kleiner Gruppen von Wissenschaftlern, sowie vor allem die Förderung junger, noch in der Ausbildung stehender Kollegen durch gezielte Integration in den Diskussionsprozeß. Unsere Gesellschaft hat das Ziel, solche und ähnliche Bestrebungen internationaler Kooperation weiterhin verstärkt zu initiieren oder zu vermitteln sowie aktiv zu unterstützen. Diesbezügliche Anregungen und Vorschläge werden gerne entgegengenommen.

Gerhard Jaritz